1. Prove the identity. **(3 marks)**

$$\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$$

Left side	Right side

Left side:

$$\frac{\sin^2\theta}{\cos^2\theta} - \frac{\sin^2\theta}{1}\,\frac{\times\cos^2\theta}{\times\cos^2\theta}$$

$$= \frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}$$

$$= \frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta}$$

$$= \frac{\sin^2\theta\sin^2\theta}{\cos^2\theta} = \frac{\sin^4\theta}{\cos^2\theta}$$

Right side:

$$\frac{\sin^2\theta}{\cos^2\theta} \times \frac{\sin^2\theta}{1}$$

$$\frac{\sin^4\theta}{\cos^2\theta}$$

Q.E.D.

2. Prove the identity. **(2 marks)**

$$\frac{\sec\theta - \cos\theta}{\tan\theta} = \sin\theta$$

Left side	Right side

Left side:

$$\frac{\left(\frac{1}{\cos\theta} - \frac{\cos\theta}{1}\right)\frac{\cos\theta}{1}}{\frac{\sin\theta}{\cos\theta}\,\frac{\cos\theta}{1}}$$

$$= \frac{1 - \cos^2\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta}{\sin\theta} = \sin\theta$$

Right side:

Q.E.D.

3. Prove the identity. **(2 marks)**

$$\frac{\cos\theta + \sin\theta\tan\theta}{\sin\theta\sec\theta} = \csc\theta$$

Left side	Right side
$\dfrac{\frac{\cos\theta}{1} + \frac{\sin\theta}{1}\,\frac{\sin\theta}{\cos\theta}}{\sin\theta \times \frac{1}{\cos\theta}}$	$\dfrac{1}{\sin\theta}$
$= \dfrac{\left(\frac{\cos\theta}{1} + \frac{\sin^2\theta}{\cos\theta}\right)}{\frac{\sin\theta}{\cos\theta}} \times \dfrac{\frac{\cos\theta}{1}}{\frac{\cos\theta}{1}}$	
$= \dfrac{\cos^2\theta + \sin^2\theta}{\sin\theta}$	
$= \dfrac{1}{\sin\theta}$	Q.E.D.

4. Prove the identity. **(2 marks)**

$$\frac{(1-\sin\theta)}{(1-\sin\theta)}\frac{1}{1+\sin\theta} = \sec^2\theta - \frac{\tan\theta}{\cos\theta}$$

Left Side	Right Side
$= \dfrac{1-\sin\theta}{1-\sin^2\theta}$	$\dfrac{1}{\cos^2\theta} - \dfrac{\frac{\sin\theta}{\cos\theta}}{\cos\theta}$
$= \dfrac{1-\sin\theta}{\cos^2\theta}$	$= \dfrac{1}{\cos^2\theta} - \dfrac{\sin\theta}{\cos^2\theta}$
	$= \dfrac{1-\sin\theta}{\cos^2\theta}$
	Q.E.D.

5. Prove the following identity: **(2 marks)**

$$\sin\theta + \cos\theta\cot\theta = \csc\theta$$

Left side	Right side
$\frac{\sin\theta}{1} + \frac{\cos\theta}{1} \times \frac{\cos\theta}{\sin\theta}$	$= \frac{1}{\sin\theta}$
$= \frac{\sin\theta \times \sin\theta}{1 \times \sin\theta} + \frac{\cos^2\theta}{\sin\theta}$	
$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta}$	
$= \frac{1}{\sin\theta}$	Q. E. D.

6. Prove the following identity: **(3 marks)**

$$\frac{(\csc\theta+1)}{(\csc\theta+1)}\frac{\cot\theta}{(\csc\theta-1)} = \frac{\csc\theta+1}{\cot\theta}$$

LEFT SIDE	RIGHT SIDE
$\dfrac{\cot\theta(\csc\theta+1)}{\csc^2\theta-\cancel{\csc\theta}+\cancel{\csc\theta}-1}$	
$=\dfrac{\cancel{\cot}\theta(\csc\theta+1)}{\cot^{\cancel{2}}\theta}$	
$=\dfrac{\csc\theta+1}{\cot\theta}$	

7. Prove the identity: **(3 marks)**

$$\frac{\sin\theta + \tan\theta}{1+\cos\theta} = \frac{\sin 2\theta}{2\cos^2\theta}$$

LEFT SIDE	RIGHT SIDE
$= \dfrac{\left(\dfrac{\sin\theta}{1} + \dfrac{\sin\theta}{\cos\theta}\right)}{\left(\dfrac{1}{1} + \dfrac{\cos\theta}{1}\right)} \times \dfrac{\dfrac{\cos\theta}{1}}{\dfrac{\cos\theta}{1}}$	$= \dfrac{\cancel{2}\sin\theta\cancel{\cos}\theta}{\cancel{2}\cos^{\cancel{2}}\theta}$
$= \dfrac{\sin\theta\cos\theta + \sin\theta}{\cos\theta(1+\cos\theta)}$	$= \dfrac{\sin\theta}{\cos\theta}$
$= \dfrac{\sin\theta\cancel{(\cos\theta+1)}}{\cos\theta\cancel{(1+\cos\theta)}}$	
$= \dfrac{\sin\theta}{\cos\theta}$	Q.E.D.

8. Prove the identity: **(2 marks)**

$$\frac{\csc\theta}{\tan\theta + \cot\theta} = \cos\theta$$

Left Side	Right Side
$= \dfrac{\frac{1}{\sin\theta}}{\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}\right)} \times \dfrac{\frac{\sin\theta\cos\theta}{1}}{\frac{\sin\theta\cos\theta}{1}}$	
$= \dfrac{\cos\theta}{\sin^2\theta + \cos^2\theta}$	
$= \dfrac{\cos\theta}{1}$	Q.E.D.

9. Prove the identity. **(2 marks)**

$$\frac{1-\cos\theta}{\sin^2\theta} = \frac{1}{1+\cos\theta}\cdot\frac{(1-\cos\theta)}{(1-\cos\theta)}$$

Left Side	Right Side
	$= \dfrac{1-\cos\theta}{1-\cancel{\cos\theta}+\cancel{\cos\theta}-\cos^2\theta}$
	$= \dfrac{1-\cos\theta}{1-\cos^2\theta}$
	$= \dfrac{1-\cos\theta}{\sin^2\theta}$
	Q.E.D.

10. Prove the identity: **(3 marks)**

$$\frac{\sin 2\theta}{\cos\theta} + \frac{\cos 2\theta}{\sin\theta} = \csc\theta$$

LEFT SIDE	RIGHT SIDE
$\frac{2\sin\theta\cancel{\cos\theta}}{\cancel{\cos\theta}} + \frac{1-2\sin^2\theta}{\sin\theta}$	$\frac{1}{\sin\theta}$
$\frac{2\sin\theta}{1} \frac{\sin\theta}{\times\sin\theta} + \frac{1-2\sin^2\theta}{\sin\theta}$	
$\frac{\cancel{2\sin^2\theta} + 1 \cancel{-2\sin^2\theta}}{\sin\theta}$	
$= \frac{1}{\sin\theta}$	Q.E.D.

11. Prove:

$$\frac{\sin 2\theta}{2 - 2\cos^2\theta} = \cot\theta \qquad \textbf{(3 marks)}$$

Left side	Right side
$\dfrac{2\sin\theta\cos\theta}{2 - 2\cos^2\theta}$	$\dfrac{\cos\theta}{\sin\theta}$
$= \dfrac{\cancel{2}\sin\theta\cos\theta}{\cancel{2}(1-\cos^2\theta)}$	
$= \dfrac{\cancel{\sin\theta}\cos\theta}{\sin^2\theta}$	
$= \dfrac{\cos\theta}{\sin\theta}$	Q.E.D.

12. Prove: **(4 marks)**

$$\frac{\sin\theta\cos\theta}{1+\cos\theta}\cdot\frac{(1-\cos\theta)}{(1-\cos\theta)} = \frac{1-\cos\theta}{\tan\theta}$$

LEFT SIDE	RIGHT SIDE
$\dfrac{\sin\theta\cos\theta-\sin\theta\cos^2\theta}{1-\cos^2\theta}$	$\dfrac{1-\cos\theta}{\frac{\sin\theta}{\cos\theta}}$
$=\dfrac{\sin\theta\cos\theta-\sin\theta\cos^2\theta}{\sin^2\theta}$	$=\left(\dfrac{1}{1}-\dfrac{\cos\theta}{1}\right)\times\dfrac{\cos\theta}{\sin\theta}$
$=\dfrac{\cancel{\sin\theta}\cos\theta(1-\cos\theta)}{\sin^{\cancel{2}}\theta}$	$=\dfrac{\cos\theta}{\sin\theta}-\dfrac{\cos^2\theta}{\sin\theta}$
$=\dfrac{\cos\theta(1-\cos\theta)}{\sin\theta}$	$=\dfrac{\cos\theta-\cos^2\theta}{\sin\theta}$
	$=\dfrac{\cos\theta(1-\cos\theta)}{\sin\theta}$

Q.E.D.!

13. Prove the identity: **(3 marks)**

$$\csc^2 x + \sec^2 x \qquad = \qquad \csc^2 x \sec^2 x$$

LEFT SIDE	RIGHT SIDE
$\frac{1}{(\sin^2 x)}\frac{(\cos^2 x)}{(\cos^2 x)} + \frac{1}{\cos^2 x}\frac{(\sin^2 x)}{\sin^2 x}$	$\frac{1}{\sin^2 x} \cdot \frac{1}{\cos^2 x}$
$= \frac{\cos^2 x + \sin^2 x}{\sin^2 x \cos^2 x}$	$= \frac{1}{\sin^2 x \cos^2 x}$
$= \frac{1}{\sin^2 x \cos^2 x}$	
	Q.E.D.

14. Prove the identity: **(3 marks)**

$$\frac{1}{\sec\theta + \tan\theta} = \frac{1-\sin\theta}{\cos\theta}$$

LEFT SIDE	RIGHT SIDE

$$\frac{1}{\left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)} \times \frac{\frac{\cos\theta}{1}}{\frac{\cos\theta}{1}}$$

$$= \frac{\cos\theta}{1+\sin\theta} \times \frac{(1-\sin\theta)}{(1-\sin\theta)}$$

$$= \frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta}$$

$$= \frac{\cancel{\cos\theta}(1-\sin\theta)}{\cos^{\cancel{2}}\theta}$$

$$= \frac{1-\sin\theta}{\cos\theta}$$

15. Prove the identity: **(3 marks)**

$$\frac{\cos 2\theta}{\sin\theta} = \frac{\cot^2\theta - 1}{\csc\theta}$$

LEFT SIDE	RIGHT SIDE
$\dfrac{\cos^2\theta - \sin^2\theta}{\sin\theta}$	$\dfrac{\dfrac{\cos^2\theta}{\sin^2\theta} - \dfrac{1}{1}}{\dfrac{1}{\sin\theta}}$
	$= \left(\dfrac{\cos^2\theta}{\sin^2\theta} - \dfrac{1}{1}\right) \times \dfrac{\sin\theta}{1}$
	$= \dfrac{\cos^2\theta}{\sin\theta} - \dfrac{\sin\theta}{1}\dfrac{(\sin\theta)}{(\sin\theta)}$
	$= \dfrac{\cos^2\theta - \sin^2\theta}{\sin\theta}$
	Q.E.D.

16. Prove the identity: **(4 marks)**

$$\frac{\cot\theta - 1}{1 - \tan\theta} = \frac{\csc\theta}{\sec\theta}$$

LEFT SIDE	RIGHT SIDE
$\dfrac{\left(\dfrac{\cos\theta}{\sin\theta} - \dfrac{1}{1}\right)\dfrac{\sin\theta\cos\theta}{1}}{\left(\dfrac{1}{1} - \dfrac{\sin\theta}{\cos\theta}\right)\dfrac{\sin\theta\cos\theta}{1}}$	$\dfrac{\dfrac{1}{\sin\theta}}{\dfrac{1}{\cos\theta}}$
$= \dfrac{\cos^2\theta - \sin\theta\cos\theta}{\sin\theta\cos\theta - \sin^2\theta}$	$= \dfrac{1}{\sin\theta} \times \dfrac{\cos\theta}{1}$
$= \dfrac{\cos\theta\cancel{(\cos\theta - \sin\theta)}}{\sin\theta\cancel{(\cos\theta - \sin\theta)}}$	$= \dfrac{\cos\theta}{\sin\theta}$
$= \dfrac{\cos\theta}{\sin\theta}$	Q.E.D.

17. Prove the identity: **(4 marks)**

$$(1-\sin\theta)(\sec\theta+\tan\theta) = \frac{1}{\sec\theta}$$

LEFT SIDE	RIGHT SIDE
$\sec\theta+\tan\theta-\sin\theta\sec\theta-\sin\theta\tan\theta$	$\cos\theta$
$=\frac{1}{\cos\theta}+\cancel{\frac{\sin\theta}{\cos\theta}}-\cancel{\frac{\sin\theta}{1}\times\frac{1}{\cos\theta}}-\frac{\sin\theta}{1}\times\frac{\sin\theta}{\cos\theta}$	
$=\frac{1}{\cos\theta}-\frac{\sin^2\theta}{\cos\theta}$	
$=\frac{1-\sin^2\theta}{\cos\theta}=\frac{\cos^2\theta}{\cos\theta}$	
$=\cos\theta$	Q.E.D!

18. Prove the identity: **(5 marks)**

$$\sin 2x(\tan x + \cot x) = 2$$

LEFT SIDE	RIGHT SIDE
$\sin 2x \tan x + \sin 2x \cot x$	
$= 2\sin x \cancel{\cos x} \cdot \frac{\sin x}{\cancel{\cos x}} + 2\cancel{\sin x}\cos x \cdot \frac{\cos x}{\cancel{\sin x}}$	
$= 2\sin^2 x + 2\cos^2 x$	
$= 2(\sin^2 x + \cos^2 x)$	
$= 2 \times 1 = \underline{\underline{2}}$	Q.E.D.

19. Prove the identity: **(5 marks)**

$$\frac{\cot\theta}{\sin\theta - \csc\theta} = -\sec\theta$$

LEFT SIDE	RIGHT SIDE
$\dfrac{\left(\dfrac{\cos\theta}{\sin\theta}\right)\dfrac{\sin\theta}{1}}{\left(\dfrac{\sin\theta}{1} - \dfrac{1}{\sin\theta}\right)\dfrac{\sin\theta}{1}}$	$-\dfrac{1}{\cos\theta}$
$\dfrac{\cos\theta}{\sin^2\theta - 1}$	
$= \dfrac{\cos\theta}{-\cos^2\theta}$	Q.E.D.
$= -\dfrac{1}{\cos\theta}$	

20. Prove: **(5 marks)**

$$\frac{\sin 2x}{1+\cos 2x} = \frac{\sec^2 x - 1}{\tan x}$$

LEFT SIDE	RIGHT SIDE
$\frac{2\sin x\cos x}{\cancel{1} + 2\cos^2 x - \cancel{1}}$	$\frac{\tan^2 x}{\tan x}$
$= \frac{\cancel{2}\sin x\cancel{\cos x}}{\cancel{2}\cos^{\cancel{2}} x}$	$= \tan x$
$= \frac{\sin x}{\cos x}$	
$= \tan x$	Q.E.D.

21. Prove: **(5 marks)**

$$\frac{2\cos x + 2\cos^2 x}{\sin 2x} = \frac{\sin x}{1-\cos x}\cdot\frac{(1+\cos x)}{(1+\cos x)}$$

LEFT SIDE	RIGHT SIDE
$= \dfrac{\cancel{2\cos x}(1+\cos x)}{\cancel{2}\sin x \cancel{\cos x}}$	$= \dfrac{\sin x(1+\cos x)}{1-\cos^2 x}$
$= \dfrac{1+\cos x}{\sin x}$	$= \dfrac{\cancel{\sin x}(1+\cos x)}{\sin^{\cancel{2}} x}$
	$= \dfrac{1+\cos x}{\sin x}$
	Q.E.D.